Forecasting Floods at Variable Catchment Scales Using Stochastic Systems Hydro-geomorphic Models and Digital Topographic Data

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Abstract This paper presents methodologies for flood forecasting using a stochastic systems hydro-geomorphic model. The model is a convolution integral of an analytical kernel with an effective input, and the kernel is an analytical geomorphic instantaneous unit hydrograph (GIUH) based on a topographically random channel network (TRCN) model with its parameter values derived from topographic data using a digital elevation model (DEM) and a geographic information system (GIS) ARC/INFO. Simulations using a stable modified midpoint method with inverse procedures and observed data show that the derived effective inputs are independent of the threshold of the topographic data from which the values of parameters are derived. These simulations endorse the high reliability of the methodologies for such applications as flood forecasting. It is found, following the analysis of topographic data from 11 small catchments in the South Island of New Zealand, that the relationships between the peak discharge and time to peak discharge of the GIUH are in perfect agreement with the theoretical values given by the model. This finding is also a test of the model and its theoretical rationale.

1. INTRODUCTION

Flood forecasting has been a central issue in civil engineering since ancient times. In modern times, efforts are continuing since the basic methods for flood forecasting stem from highly simplified methods. Uncertainties in flood forecasting are yet to be clarified since the basic methods in use today lack a concrete theoretical basis. The TRCN model is one of the recent advances in modelling flows in topographic channel networks based on the GIUH concept [Rodriguez-Iturbe and Valdés, 1979]. This model links the hydrologic response of a catchment to geomorphologic parameters. One of the most significant advances in this area is the link-based model developed by Troutman and Karlinger [TK model] [1984; 1985] which incorporates the concept of TRCN and all pertinent hydraulic information. In this paper, we present a stochastic systems hydro-geomorphic model for routing flows in TRCN which incorporates the rigorously derived analytical TK model as a kernel with explicit topographic and hydraulic parameters evaluated using DEM and GIS ARC/INFO GRID hydrological functions. Simulations using an inverse problem of the model [Liu *et al.*, 1997] indicate that this model is robust, stable and reliable, and that its computed outputs or inversely derived inputs are independent of its hydraulic and topographic parameters. These features are valuable features for flood forecasting and therefore we turn our attention to introducing the model and the numerical procedures to flood forecasting.

2. THE STOCHASTIC SYSTEMS HYDRO-GEOMORPHIC MODEL

In systems hydrology terms, the output from a system is a convolution integral of an impulse response (kernel) with the input to the system [Chow *et al.*, 1988], viz.

$$Q(t) = \int_0^t I(\tau)\bar{u}(t-\tau)d\tau \qquad (1)$$

where

$Q_{(t)}$ is the outflow;

I is the input function;

$\bar{u}$ is the kernel, which is a stochastic impulse response represented by a GIUH;

t is time; and

τ is a time lag.

In the traditional method, a discrete form of (1) is used with a derived unit hydrograph for the kernel to compute the outflow [Chow *et al.*, 1988]. Chapman [1985] used polynomial functions to approximate both the input and the kernel in (1). This approach is useful but one cannot interpret the physical meaning of the parameters since the polynomials do not have physical entities. Jakeman *et al.* [1990] used a Simple Refined Instrumental Variables (SRIV) algorithm to achieve several important goals including baseflow separation as an integral part of model identification with short data sets. In this paper, instead of identifying the model and its parameters, we present a model with an analytical GIUH for the kernel and the values of its parameters derived from topographic data using GIS and DEM.

2.1. The Basic Framework of the Model Based on Network Hydro-geomorphology

For flows in topologically random channel networks with fundamental basin characteristics, Troutman and Karlinger [1984; 1985] developed the following model,

$$\bar{u}_Z(t,\alpha,\beta) = 1 - \int_0^t \bar{h}(\tau:\alpha,\beta)d\tau \quad +$$

$$2\sum_{z_1} P[Z_1 = z_1 | Z = z]\int_0^t \bar{h}(t-\tau:\alpha;\beta)\bar{u}_{z_1}(\tau:\alpha:\beta)d\tau \qquad (2)$$

where

$\bar{u}_{z_1}(t,\alpha,\beta)$ is the expectation of the routed discharge, u;

$\bar{h}$ is the impulse response averaged with respect to the link length distribution F;

P is the probability with the fundamental basin characteristics Z;

α is the mean internal link length of the channel segment;

β is a parameter having two meanings: $\beta = 1.0$ for a width function; or $\beta = 1.5V_0$ for celerity with V_0 the equilibrium velocity of channel flow.

"The key idea in obtaining ..." (2) [(9) and therefore (11), in Troutman and Karlinger, 1985, p. 753] ... "is to recognise that the two upstream links joining the outlet link at this junction may themselves be taken to be the outlets of two subnetworks." For the detailed derivation and nomenclature the reader is referred to Troutman and Karlinger [1984; 1985].

With (2) for an arbitrary distribution of the link length in a network, the asymptotic results for three linear routing methods are given as three distribution functions: (A) a Weibull distribution for a translation routing; (B) a beta distribution for a diffusion routing, and (C) a uniform distribution for a general routing. A Weibull distribution is used here, viz.

$$\bar{u}(t) = \frac{\lambda^2 t}{2n} exp\left[-\left(\frac{\lambda t}{2\sqrt{n}}\right)^2\right] \qquad t \geq 0 \qquad (3)$$

where

$\bar{u}$ is the topologically expected IUH as a function of time;

$$\lambda = \frac{\beta}{\alpha}; \qquad (4)$$

and

n is the number of first-order streams.

Since the asymptotic solution of the TK model is an expression for the IUH, (3) is a kernel in (1), which, using the change of variables $x = t - \tau$, is written as

$$Q_{(t)} = \int_0^t I_{(t-x)} \frac{\lambda^2 x}{2n} exp\left[-\left(\frac{\lambda x}{2\sqrt{n}}\right)^2\right] dx \qquad (5)$$

(5) is no longer the traditional convolution integral. Its present kernel now includes explicit hydro-geomorphic parameters, and in flood forecasting, its input is the design rainfall and its output is the flood generated by the rainfall.

The time to the peak discharge of the unit hydrograph (UH), t_p, is given by

$$t_p = \frac{(2n)^{1/2}}{\lambda} \qquad (6)$$

and the peak discharge of UH, U_p, is given by

$$U_p = \frac{\lambda}{(2ne)^{1/2}} \qquad (7)$$

where

$e = 2.71828.$

(6) and (7) are combined to yield

$$U_p = \frac{0.6065}{t_p} \qquad (8)$$

which is very useful for examining the relationships between the two important parameters of a hydrograph.

2.2. Numerical Scheme for the Model

For numerical solutions of (5), several methods have been developed. Linz [1969] showed that direct methods which replace the integral by either the rectangular midpoint quadrature or the trapezoidal quadrature are stable while many direct methods which replace the integral by higher order quadratures are not convergent [Anderssen and White, 1971]. As the midpoint method is numerically stable, we use it to discretise (5).

The numerical solutions of (5) exist under the following conditions:

(i) $Q_{(t)}$ is continuously differentiable on $0 \leq t \leq \tau$;

(ii) $Q_{(0)} = 0$;

(iii) $\bar{u}_{(t-\tau)}$ and $\frac{\partial \bar{u}_{(t-\tau)}}{\partial t}$ are continuous on $0 \leq \tau \leq t \leq T$; and

(iv) $\bar{u}_{(0)} \neq 0$.

Under the above conditions, (5) has a unique continuous solution $I(t)$ on $0 \leq t \leq T$ [de Hoog and Weiss, 1973], and is solved in the interval $[0,T]$ by dividing the interval into smaller intervals of width Δx, the i-th point of subdivision being denoted by x_i, such that $x_i = i\Delta x$, $i=0, 1, \ldots, N$, and $N\Delta x = T$. The solution is approximated by its value at the point

$$x = x_{i+1/2} = x_i + \frac{\Delta x}{2} \qquad (9)$$

With (9), expanding and summarising (5), using the product integration to separate the kernel, gives

$$Q_{(t_n)} = \sum_{i=0}^{n-1} I(t_n - x_{i+1/2}) \int_{x_i}^{x_{i+1}} \bar{u}_{(x)} dx \qquad (10)$$

or

$$Q_{(t_n)} = \sum_{i=0}^{n-1} I_{n-i-1/2} A_i \qquad (11)$$

with

$$A_i = \int_{x_i}^{x_{i+1}} \bar{u}_{(x)} dx \qquad (12)$$

which is equivalent to

$$A_i = \int_{x_i}^{x_{i+1}} \left(\frac{\lambda^2 x}{2n}\right) exp\left(\frac{\lambda^2 x^2}{4n}\right) dx \qquad (13)$$

which is integrated, and combined with (11) to yield,

$$Q(t_n) = \sum_{i=0}^{n-1} I_{n-i-1/2} - exp\left(-\frac{\lambda^2 \Delta x^2}{4n}\right)$$
$$\cdot\left\{exp\left[-(i+1)^2\right] - exp(-i^2)\right\} \qquad (14)$$

which is the convolution integral (5) in a discrete form with a kernel given by (3).

3. PARAMETER VALUES DERIVED FROM TOPOGRAPHIC DATA

Eleven small catchments have been analysed for the topographic parameters. A table below gives a brief description of the catchments.

Table 1. Eleven small catchments used for analysis

Names	Abbreviation	Area (ha)	Location
Glendhu	G1	218.0	Otago
Glendhu	G2	310.0	Otago
Donald Creek	DC1	8.57	Nelson
Donald Creek	DC2	4.77	Nelson
Donald Creek	DC3	7.84	Nelson
Donald Creek	DC4	20.19	Nelson
Maimai	M5	2.31	W. Coast
Maimai	M6	1.63	W. Coast
Maimai	M8	3.84	W. Coast
Maimai	M13	4.25	W. Coast
Maimai	M14	4.62	W. Coast

The values of model parameters were derived using GIS ARC/INFO GRID and hydrological functions working on a 2m DEM grid. The DEM was generated from 5m contour data using a DEM interpolator developed in-house by Landcare Research New Zealand Ltd, referred to here as the Giltrap method [D. Giltrap, pers. comm.]. The initial DEM was subjected to several iterations of the ARC GRID FILL command to create a "depressionless" DEM and hence ensure continuity of drainage over the terrain surface [Jensen and Domingue, 1991]. This DEM surface was then analysed hydrologically (i.e., for FLOWDIRECTION and FLOWACCUMULATION) to build a series of stream networks based on a range of thresholds for channelised flow [e.g., 100 to 1,000 cells]. The raster cell networks generated by this process were analysed for stream order (i.e., using STREAMORDER) and converted to an ARC/INFO vector coverage (i.e, streamline) with each stream segment having attributes including stream order and length. Analysis of the arc attribute tables for these network coverages yielded statistics for the number of first-order stream segments (n), and for the mean internal length of channel segments (α) for each network which is the mean segment length of all stream orders excluding first order streams.

Figure 1 below shows the variation of parameters n and α with threshold for catchment G1. The threshold refers to the number of 2m DEM cells required to initiate channel flow.

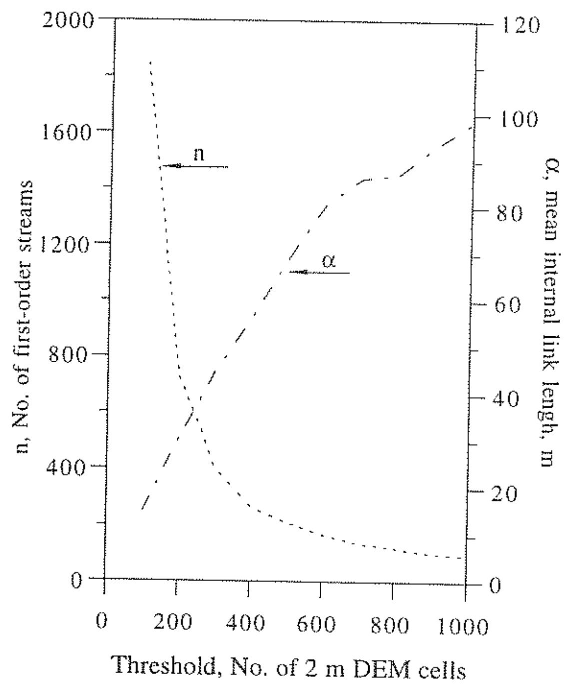

Fig. 1. Parameters n and α vary with threshold

These two parameters are used in computing the temporal process of a hydrograph and the peak discharge and time to peak of the hydrograph.

4. FLOOD FORECASTING USING THE MODEL AND MODEL TEST

In the following example, we use the model for flood forecasting. The rainfall pattern of March 1995 in Glendhu catchment is increased by 15 times to be a "design storm", and the computed "design flood" hydrographs are shown in Fig. 2.

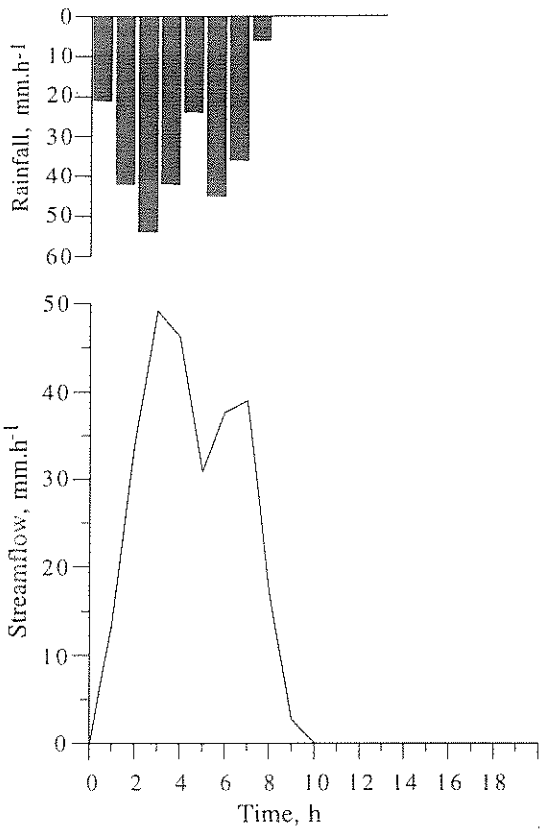

Fig. 2. Design storm and corresponding streamflow, Glendhu catchment 1, New Zealand

Furthermore, let us briefly discuss the model test. There are several ways to test the validity of the model. One of these approaches is to examine the theoretical values of the parameters against the observed ones. Figure 3 shows a testing of the relationships between the peak discharge and time to peak of the GIUH as indicated by [8].

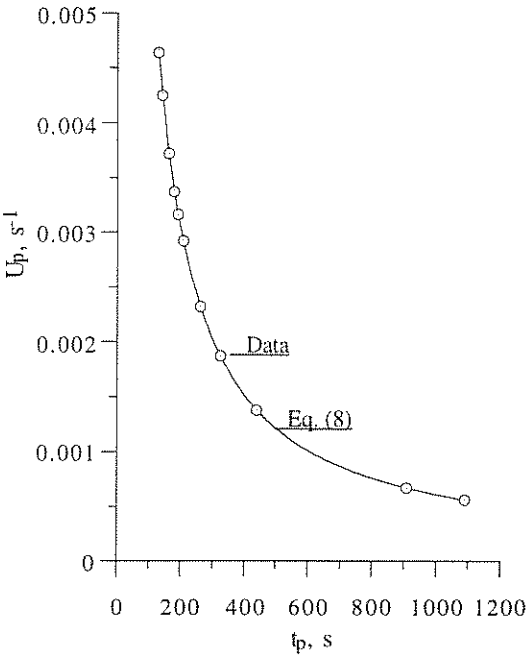

Fig. 3. Theoretical and observed values of U_p and t_p for 11 catchments in New Zealand

It is seem from Fig. 3 that the theoretical values of U_p and t_p for the 11 catchments are in perfect agreement with the observed ones. This perfect fitting implies that the TRCN theory and rationale for developing the model, and the procedures used for deriving the values of the model parameters are justified. This fitting is also a good test of the model.

5. CONCLUSIONS AND DISCUSSIONS

[1]. A systems hydro-geomorphic model with an analytical kernel is proposed and demonstrated to compute the outflow hydrographs for flood forecasting. A stable numerical method, the modified midpoint method, is used for discretising the model, and GIS ARC/INFO algorithms are used to derive the values of the parameters from digital topographic data.

[2]. Following the analysis of topographic data from 11 small catchments it is found that the relationships between the peak discharge and

time to peak discharge of the GIUH are in perfect agreement with the theoretical values given by the model. This result endorses the robust features of the model and theoretical rationale. It is, therefore, clear that this model is very suitable for practical applications.

[3]. Since the topographic and hydraulic parameters of the kernel can be determined using GIS and DEM, the procedures offer a valuable tool for forecasting floods in ungauged regions. Because of the scale and resolution independence, topographic data with different resolutions and scales would satisfy the same requirements for forecasting floods using this model.

[4]. We have shown that hydro-geomorphic models and topographic data can provide a concrete basis for flood forecasting. We recommend that the model, with its parameters derived in similar ways, be used for modelling solute and sediment transport at catchment scales, since the TK model is applicable in theory to solute and sediment transport processes in the catchment networks.

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